

A BAYESIAN APPROACH TO OUTPUT GAP ESTIMATION ^(*)

C. Planas¹, A. Rossi¹, G. Fiorentini²

¹ Joint Research Centre, European Commission, Ispra (VA)

² University of Florence, Department of Statistics, Florence (FI)

e-mail: christophe.planas@jrc.it

Riassunto: In questo lavoro si derivano stimatori del prodotto potenziale che incorporano le informazioni derivanti dalla curva di Phillips e dalla teoria del ciclo economico. A tale scopo si sviluppa un'analisi Bayesiana del modello bivariato di Kuttner (1994). La metodologia proposta si applica alle serie storiche degli U.S. e dell'area Euro.

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1. Introduction

In this paper we develop a Bayesian analysis of the Phillips-curve bivariate model put forward by Kuttner (1994) for estimating potential output and output gap. Kuttner (1994) original model relates output gap to inflation through the Phillips curve.

In this context of cycle estimation embedded into a Phillips curve regression, an important amount of macroeconomic theory and business cycle knowledge is available. Bayesian analysis allow us to incorporate this information into the decomposition. This yields several benefits. First, the finite-sample uncertainty around any quantity of interest can be delineated. Second, the insertion of additional knowledge is expected to make Bayesian gap estimates more precise than maximum likelihood ones. Third, the Bayesian setting helps in understanding salient features of Phillips curve regressions. Finally, by properly tuning the prior distribution of variance parameters the pile-up problem sometimes faced in classical analysis can be avoided. Given priors on model parameters, we implement a Gibbs sampling scheme for drawing model parameters and state vector from their joint posterior distribution.

2. Model specification

Let y_t denote (log) real output. Like Watson (1986) and Clark (1987), we assume that it is made up of a trend, p_t , and of a cycle, c_t , according to:

$$y_t = p_t + c_t; \quad \Delta p_t = \mu_p + a_{pt}; \quad \phi_c(L)c_t = a_{ct} \quad (2.1)$$

where L is the lag operator, Δ represents the first-difference, μ_p is a constant drift and $\phi_c(L) = 1 - \phi_{c1}L - \phi_{c2}L^2$. The shocks, a_{pt} and a_{ct} , are orthogonal Gaussian white

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noises with variances V_p and V_c . The long-term and short-term components, p_t and c_t , are interpreted as potential output and output gap.

Kuttner (1994) complemented (2.1) with an equation that dynamically links change in inflation, say $\Delta\pi_t$, to the output gap like in:

$$\phi_\pi(L)\Delta\pi_t = \mu_\pi + \beta c_{t-1} + \lambda \Delta y_{t-1} + a_{\pi t}^*, \quad (2.2)$$

where $a_{\pi t}^*$ is a Gaussian white noise and the roots of the AR(2) polynomial $\phi_\pi(L) = 1 - \phi_{\pi 1}L - \phi_{\pi 2}L^2$ are assumed stationary. Equation (2.2) introduces a Phillips curve effect by relating the change in inflation to the real output growth and to the latent cycle, both with one lag. The innovations in inflation and in the gap are assumed contemporaneously correlated, with correlation coefficient $\rho_{c\pi}$. Let κ_π denote a real parameter and $a_{\pi t}$ be a Gaussian white noise with variance V_π orthogonal to a_{ct} . We reparameterize $\rho_{c\pi}$ by writing the innovations $a_{\pi t}^*$ as $a_{\pi t}^* \equiv \kappa_\pi a_{ct} + a_{\pi t}$, so (2.2) becomes:

$$\phi_\pi(L)\Delta\pi_t = \mu_\pi + \beta c_{t-1} + \lambda \Delta y_{t-1} + \kappa_\pi a_{ct} + a_{\pi t}, \quad (2.3)$$

and $\rho_{c\pi} = \kappa_\pi \sqrt{V_c} / \sqrt{\kappa_\pi^2 V_c + V_\pi}$. We reparameterize the model for the cycle as in:

$$(1 - 2A \cos(2\pi/\tau) L + A^2 L^2) c_t = a_{ct} \quad (2.4)$$

where the parameters A and τ represent the amplitude and periodicity.

The model is completed by specifying the prior distributions. Let $\delta = (\mu_\pi, \beta, \lambda, \phi_{\pi 1}, \phi_{\pi 2}, \kappa_\pi)$ denote the vector of conditional mean parameters in equation (2.3). We shall consider the block-independence structure:

$$p(A, \tau, V_c, \delta, V_\pi, \mu_p, V_p) = p(A)p(\tau)p(V_c)p(\delta, V_\pi)p(\mu_p, V_p)$$

with

$$\begin{aligned} p(A) &= \text{Beta}(a_A, b_A); & p\left(\frac{\tau - \tau_l}{\tau_u - \tau_l}\right) &= \text{Beta}(a_\tau, b_\tau); & p(V_c) &= IG(s_{c0}, v_{c0}) \\ p(\delta, V_\pi) &= NIG(\delta_0, M_\delta^{-1}, s_{\pi 0}, v_{\pi 0})I_\delta; & p(\mu_p, V_p) &= NIG(\mu_{p0}, M_{p0}^{-1}, s_{p0}, v_{p0}) \end{aligned} \quad (2.5)$$

where $\text{Beta}(\cdot, \cdot)$ is the Beta p.d.f., τ_l and τ_u are the lower and upper bounds of τ 's support, $IG(\cdot)$ is the inverted Gamma distribution, $NIG(\cdot)$ is the Normal-inverted Gamma distribution and I_δ is an index set for imposing constraints on the support.

Let θ denote the full set of parameters, i.e. $\theta \equiv (A, \tau, V_c, \delta, V_\pi, \mu_p, V_p)$. Our objective is to characterize the joint posterior distribution of potential output, output gap and of the parameters conditionally on the data, i.e. $p(c^T, p^T, \theta | Y^T)$ where $x_k^T \equiv \{x_k, \dots, x_T\}$, $x^T \equiv x_1^T$ and $Y^T \equiv \{y^T, \Delta\pi^T\}$. Given our model, no closed form expression for this posterior is available but draws from $p(c^T, p^T, \theta | Y^T)$ can be obtained using a Gibbs sampling scheme. The full conditionals of interest are:

$$p(c^T, p^T | \theta, Y^T); \quad p(\theta | c^T, p^T, Y^T).$$

3. Joint posterior distribution of states and model parameters

To simulate the components conditionally on model parameters, it is useful to cast equations (2.1), (2.3) and (2.4) into a state space form:

$$Y_t = H\xi_t; \quad \xi_{t+1} = D + F\xi_t + w_{t+1},$$

where $Y_t = (y_t, \pi_t)'$ is the vector of observations, $\xi_t = (p_t, c_t, c_{t-1}, \kappa_\pi a_{ct} + a_{\pi t})'$ is the state vector, $w_t = (a_{pt}, a_{ct}, 0, \kappa_\pi a_{ct} + a_{\pi t})'$ is a Gaussian error vector with zero mean and singular variance matrix Q . The time-invariant matrices H , D , F and Q can be straightforwardly recovered. Like usual, $\xi_{t|k}$ and $P_{t|k}$ denote the conditional expectation $E(\xi_t|Y^k)$ and variance $Var(\xi_t|Y^k)$. Samples from $p(c^T, p^T|\theta, Y^T)$ will be obtained through $p(\xi^T|\theta, Y^T)$. We make use of the following identity that gives the basis of the Carter and Kohn (1994) state sampler:

$$p(\xi^T|\theta, Y^T) = p(\xi_T|\theta, Y_T) \prod_{t=2}^{T-1} p(\xi_t|\theta, Y^t, \xi_{t+1}) p(\xi_1|\theta, Y_1, \xi_2)$$

where the last term is isolated because initial conditions need a special treatment. A draw from $p(\xi^T|\theta, Y^T)$ can be obtained as follows:

- (i) compute $\xi_{t|t}$, and $P_{t|t}$, $t = 2, \dots, T$, via the diffuse Kalman filter (de Jong, 1991);
- (ii) given $\xi_{T|T}$ and $P_{T|T}$, sample ξ_T from $p(\xi_T|\theta, Y^T) = N(\xi_{T|T}, P_{T|T})$;
- (iii) for $t = T - 1$ to $t = 2$, sample backward ξ_t from $p(\xi_t|\theta, \xi_{t+1}, Y^t)$.
- (iv) sample ξ_1 from $p(\xi_1|\theta, \xi_2, Y_1) = N(E[\xi_1|\theta, \xi_2, Y_1], V[\xi_1|\theta, \xi_2, Y_1])$.

Not all elements of the state need to be simulated. Trivially, given y_t knowledge of c_t determines p_t . Also, given model parameters and observations up to time t , sampling c_{t-1} yields the last state's element, $\kappa_\pi a_{ct} + a_{\pi t}$, through equation (2.3).

We now turn to the second full conditional distribution, $p(\theta|c^T, p^T, Y^T) = p(\theta|c^T, p^T, \Delta\pi^T)$. Several approaches are possible: the strategy we put forward builds three parameter blocks. Indeed the model implies that the density $p(p^T, c^T, \Delta\pi^T|\theta)$ can be factorized as:

$$\begin{aligned} p(p^T, c^T, \Delta\pi^T|\mu_p, V_p, A, \tau, V_c, \delta, V_\pi) &= \\ &= p(p^T|\mu_p, V_p) p(c^T|A, \tau, V_c) p(\Delta\pi^T|c^T, p^T, A, \tau, V_c, \delta, V_\pi, \mu_p, V_p) \end{aligned}$$

Then the block-independence assumption about the parameter priors makes the full conditional $p(\theta|p^T, c^T, Y^T)$ verifying:

$$\begin{aligned} p(\mu_p, V_p, A, \tau, V_c, \delta, V_\pi|c^T, p^T, \Delta\pi^T) &= \\ &= p(\mu_p, V_p|p^T) p(A, \tau, V_c|c^T) p(\delta, V_\pi|c^T, p^T, \Delta\pi^T, A, \tau, V_c, \mu_p, V_p) \end{aligned}$$

We first consider the conditional $p(\mu_p, V_p|p^T)$. Choosing the conjugate Normal-inverse Gamma prior yields:

$$p(\mu_p, V_p|p^T) = NIG(\mu_{p*}, M_{p*}^{-1}, s_{p*}, v_{p*})$$

where

$$\begin{aligned}
M_{p*} &= M_{p0} + T - 1 \\
\mu_{p*} &= M_{p*}^{-1}[M_{p0}\mu_{p0} + (T - 1)\hat{\mu}_p] \\
v_{p*} &= v_{p0} + T - 1 \\
s_{p*} &= s_{p0} + \left(\frac{1}{M_{p0}} + \frac{1}{T - 1}\right)^{-1}(\mu_{p0} - \hat{\mu}_p)^2 + \sum_{t=2}^T (\Delta p_t)^2 - (T - 1)\hat{\mu}_p^2
\end{aligned}$$

and $\hat{\mu}_p = \frac{1}{T-1} \sum_{t=2}^T \Delta p_t$.

Focusing next on $p(A, \tau, V_c | c^T)$, we consider the full conditionals $p(A | \tau, V_c, c^T)$, $p(\tau | A, V_c, c^T)$ and $p(V_c | A, \tau, c^T)$. The first two verify:

$$p(A | \tau, V_c, c^T) \propto p(c_1, c_2 | A, \tau, V_c) \prod_{t=3}^T p(c_t | c^{t-1}, A, \tau, V_c) p(A)$$

and

$$p(\tau | A, V_c, c^T) \propto p(c_1, c_2 | A, \tau, V_c) \prod_{t=3}^T p(c_t | c^{t-1}, A, \tau, V_c) p(\tau)$$

Sampling directly from the conditionals above is not possible but both densities are straightforward to evaluate. We use the adaptive rejection Metropolis scheme (ARMS) proposed by Gilks et al. (1995, 1997). For the distribution $p(V_c | A, \tau, c^T)$, the NIG-framework implies (see Bauwens et al., 1999, p.304):

$$p(V_c | A, \tau, c^T) = IG(s_{c*}, v_{c*})$$

with:

$$\begin{aligned}
s_{c*} &= s_{c0} + (c_1 \ c_2) \Sigma_c^{-1} (c_1 \ c_2)' + \sum_{t=3}^T a_{ct}^2 \\
v_{c*} &= v_{c0} + T
\end{aligned}$$

where Σ_c is the variance-covariance matrix of (c_1, c_2) given A and τ re-scaled by the innovation variance V_c , i.e. $\Sigma_c \equiv V[(c_1 c_2) | A, \tau, V_c] / V_c$.

It remains to sample from the full conditional $p(\delta, V_\pi | c^T, p^T, \Delta \pi^T, A, \tau, V_c, \mu_p, V_p)$:

$$p(\delta, V_\pi | c^T, p^T, \Delta \pi^T, A, \tau, V_c, \mu_p, V_p) \propto p(\Delta \pi_1, \Delta \pi_2 | c^T, p^T, \theta) \prod_{t=3}^T p(\Delta \pi_t | \Delta \pi^{t-1}, \theta) p(\delta, V_\pi)$$

Let Z denote the $(T - 2) \times 6$ matrix of regressors in equation (2.2) and $\hat{\delta}$ be the OLS estimate of δ . Given our priors, standard results in Bayesian regression analysis yield:

$$\prod_{t=3}^T p(\Delta \pi_t | \Delta \pi^{t-1}, c^T, p^T, \theta) p(\delta, V_\pi) \propto NIG(\delta_*, s_{\pi*}, M_{\pi*}, v_{\pi*}) I_\delta$$

where

$$\begin{aligned}
M_{\pi*} &= M_{\pi 0} + Z'Z \\
\delta_* &= M_{\pi*}^{-1}[M_{\pi 0}\delta_0 + Z'Z\hat{\delta}] \\
v_{\pi*} &= v_{\pi 0} + T - 2 \\
s_{\pi*} &= s_{\pi 0} + \Delta\pi_3^{T'}(I_{T-2} - Z(Z'Z)^{-1}Z')\Delta\pi_3^T + \\
&\quad + (\delta_0 - \hat{\delta})'[M_{\pi 0}^{-1} + (Z'Z)^{-1}]^{-1}(\delta_0 - \hat{\delta})
\end{aligned} \tag{3.1}$$

This result enables us to get draws from $p(\delta, V_\pi | c^T, p^T, \Delta\pi^T, A, \tau, V_c, V_p, \mu_p)$ through a Metropolis-Hastings scheme with the $NIG(\delta_*, s_{\pi*}, M_{\pi*}, v_{\pi*})I_\delta$ as proposal. For each candidate say $(\tilde{\delta}, \tilde{V}_\pi)$, the acceptance probability is given by

$$\min\left\{1, \frac{p(\Delta\pi_1, \Delta\pi_2 | \tilde{\delta}, \tilde{V}_\pi, c^T, p^T, A, \tau, V_c, \mu_p, V_p)}{p(\Delta\pi_1, \Delta\pi_2 | \delta, V_\pi, c^T, p^T, A, \tau, V_c, \mu_p, V_p)}\right\} \tag{3.2}$$

The M-H step (3.2) removes the conditioning on the starting values $\Delta\pi_1$ and $\Delta\pi_2$.

4. Model extension: the variance moderation case

Few years after the publication of Kuttner's paper, several researchers pointed out a substantial reduction of the US output growth volatility during the mid-1980's; see for instance Kim and Nelson (1999), McConnell and Perez-Quiros (2000) and the discussion in Stock and Watson (2002). Several explanations for this shrinking of the US business cycle have been proposed; interested readers are referred to Stock and Watson (2002). An easy way to insert the variance moderation into the model is to let the innovation variances verify:

$$V_{mt} = \begin{cases} V_m & t < t_B \\ \alpha_m V_m & t \geq t_B \end{cases} \quad m = p, c, \pi. \tag{4.1}$$

The time index t_B refers to the date of the change in the variance parameters that is supposed to be known; for the US economy, the first quarter of 1984 is typically retained. Equation (4.1) introduces three additional parameters, α_p , α_c and α_π , for which we assume the independent priors:

$$\alpha_m \sim \text{Beta}(a_{\alpha_m}, b_{\alpha_m}) \quad m = p, c, \pi.$$

The support of these distributions is taken as $(0, 1)$ since we expect a variance reduction. Sampling the state given parameters can be made like in Section 3.1. For sampling parameters given the state, the independence assumption on the α 's prior distribution preserves the three-block structure, the blocks of interest becoming $p(\mu_p, V_p, \alpha_p | p^T)$, $p(A, \tau, V_c, \alpha_c | c^T)$ and $p(\delta, V_\pi, \alpha_\pi | p^T, c^T, \pi^T, \mu_p, V_p, A, \tau, \alpha_p, \alpha_c)$.

Let us first focus on the trend parameters. The full conditional $p(\mu_p, V_p, \alpha_p | p^T)$ can be factorized according to:

$$p(\mu_p, V_p, \alpha_p | \Delta p^T) = p(\mu_p, V_p | \alpha_p, \Delta p^T) \times p(\alpha_p | \Delta p^T)$$

where the second term verifies:

$$p(\alpha_p|\Delta p^T) \propto p(\Delta p_{t_B}^T|\Delta p_2^{t_B-1}, \alpha_p)p(\alpha_p)$$

since $\Delta p_2^{t_B-1}$ does not depend on α_p . Let I_t denote the identity matrix of dimension t , $1_{t,k}$ a $(t \times k)$ matrix of ones and, for simplifying exposition, $1_t \equiv 1_{t,t}$. The NIG-prior assumption (2.5) implies:

$$\begin{matrix} \Delta p_2^{t_B-1} \\ \Delta p_{t_B}^T \end{matrix} | \alpha_p, V_p \sim N \left(\begin{matrix} \mu_{p0} 1_{t_B-2,1} \\ \mu_{p0} 1_{T-t_B+1,1} \end{matrix}, V_p \begin{bmatrix} I_{t_B-2} + \frac{1_{t_B-2}}{M_{p0}} & \frac{1_{t_B-2, T-t_B+1}}{M_{p0}} \\ \alpha_p I_{T-t_B+1} + \frac{1_{T-t_B+1}}{M_{p0}} \end{bmatrix} \right) \quad (4.2)$$

Conditioning over $\Delta p_2^{t_B-1}$ and α_p and marginalizing with respect to V_p yields a multivariate Student with parameters (see Bauwens et al. 1999, p.304) :

$$\Delta p_{t_B}^T | \Delta p_2^{t_B-1}, \alpha_p \sim t(E[\Delta p_{t_B}^T | \Delta p_2^{t_B-1}, \alpha_p], \{V[\Delta p_{t_B}^T | \Delta p_2^{t_B-1}, \alpha_p]\}^{-1}, s_{p*}, v_{p0} + t_B - 2)$$

where

$$s_{p*} = s_{p0} + (\Delta p_2^{t_B-1} - \mu_{p0} 1_{t_B-2,1})'(I_{t_B-2} + M_{p0}^{-1} 1_{t_B-2})^{-1}(\Delta p_2^{t_B-1} - \mu_{p0} 1_{t_B-2,1})$$

and $E[\Delta p_{t_B}^T | \Delta p_2^{t_B-1}, \alpha_p] = E[\Delta p_{t_B}^T | \Delta p_2^{t_B-1}, \alpha_p, V_p]$ and $V[\Delta p_{t_B}^T | \Delta p_2^{t_B-1}, \alpha_p] = V[\Delta p_{t_B}^T | \Delta p_2^{t_B-1}, \alpha_p, V_p]/V_p$. Both moments are straightforwardly available from the joint Normal distribution (4.2). Use can then be made of ARMS for sampling α_p with the kernel of $p(\alpha_p|\Delta p^T)$ evaluated as the product of this multivariate Student times the Beta-prior on α_p . Next, given α_p and Δp^T , samples of μ_p and V_p can be obtained similarly to Section 3.2 after re-scaling the trend equation (2.1) by $\sqrt{\alpha_p}$.

For the cycle parameters, we consider the Gibbs scheme $p(A|\tau, V_c, \alpha_c, c^T)$, $p(\tau|A, V_c, \alpha_c, c^T)$ and $p(V_c, \alpha_c|A, \tau, c^T)$. Samples of A and τ from the first two conditionals can be obtained like in Section 3 with a correction of the likelihood of c^T to account for the variance change. The factorization:

$$p(V_c, \alpha_c|A, \tau, c^T) = p(V_c|\alpha_c, A, \tau, c^T)p(\alpha_c|A, \tau, c^T)$$

shows that V_c can also be sampled like in Section 3.2 using residuals re-scaled by $\sqrt{\alpha_c}$. The sampling of α_c requires instead more attention. A simple solution is to use ARMS with the kernel of $p(\alpha_c|\tau, A, c^T)$ evaluated as $p(\alpha_c|\tau, A, c^T) \propto p(c_{t_B}^T|c^{t_B-1}, A, \tau, \alpha_c) \times p(\alpha_c)$ where, from the NIG-framework:

$$p(c_{t_B}^T|c^{t_B-1}, A, \tau, \alpha_c) \sim t(0, I_{T-t_B}/\alpha_c, s_{c*}, v_{c0} + t_B - 1)$$

with $s_{c*} = s_{c0} + (c_1, c_2)'\Sigma_c^{-1}(c_1, c_2) + \sum_{t=3}^{t_B-1} a_{ct}^2$.

To sample the Phillips curve parameters we condition on α_π in a further Gibbs-step $p(\delta, V_\pi|\alpha_\pi, p^T, c^T, \Delta\pi^T, \mu_p, V_p, A, \tau, \alpha_p, \alpha_c)$ and $p(\alpha_\pi|\delta, V_\pi, p^T, c^T, \Delta\pi^T, \mu_p, V_p, A, \tau, \alpha_p, \alpha_c)$. Re-scaling the Phillips curve equation (2.3) by $\sqrt{\alpha_\pi}$ lets the sampling of δ and V_π unchanged. Finally the sampling of α_π given all other quantities can be made with ARMS by observing that

$$p(\alpha_\pi|\delta, V_\pi, p^T, c^T, \Delta\pi^T, \mu_p, V_p, A, \tau, \alpha_p, \alpha_c) \propto p(a_{\pi t_B}, \dots, a_{\pi T}|V_\pi, \alpha_\pi)p(\alpha_\pi)$$

where the first term is the Normal distribution with mean zero mean and variance-covariance matrix $V_\pi \alpha_\pi I_{T-t_B}$.

5. Application

We now illustrate our methodology with a Bayesian investigation of Phillips-curve-based output gap in US and in the euro area. For both cases the sample is made up of 141 quarterly observations from 1970-3 to 2005-3. Inflation rate is measured on the basis of the consumer price index. An additive outlier was detected in the EU-12 CPI series at the date 1975-Q1. We thus added a dummy variable to equation (2.3).

For setting the hyperparameters of the prior distributions we consider the information available from macroeconomic theory and from business cycle knowledge. For the US case, we take into account Kuttner's results that characterize fluctuations in US economy as occurring with a 5-year recurrence and with a contraction factor around 0.8. For EU-12, we consider the results in Gerlach and Smets (1999) that describe an euro-area cycle with contraction factor close to the one of US cycles but with a longer periodicity, namely about 8 years. We center the first moment of the τ and A prior distributions on these values without imposing too much precision. Specifically, our priors are $(\tau - 2)/(141 - 2) \sim \text{Beta}(2.44, 16.35)$ and $A \sim \text{Beta}(5.82, 2.45)$ for US and $(\tau - 2)/(141 - 2) \sim \text{Beta}(2.96, 10.70)$ and $A \sim \text{Beta}(5.82, 2.45)$ for EMU. Next, we tune the prior distribution on the trend drift in order to reproduce an annual mean growth rate of about 3% for US and 2.5% for EMU, with an associated variance that allows for a annual deviation of roughly 1.5 percentage point. The prior distribution of the cycle innovation variance has been set so as to add a further 2.0 percentage points deviation on the annual growth. For US, we tune the prior on the scale parameters α_c and α_p so as to get an expected value about 0.3 with associated standard deviation 0.15. More flexibility is let to the parameter α_π . We set the break date at 1984-Q1, in agreement with Kim and Nelson (1999) and McConnell and Perez-Quiros (2000).

Prior information is also available for the Phillips curve. In particular, inflation is expected to react positively to the lagged output gap and to the lagged output growth. The contemporaneous correlation between shocks in gap and shocks in inflation should also be positive. The prior densities of all quantities of interest are depicted in figures 1-3.

We obtain draws from the joint posterior distribution of gap and model parameters following the MCMC scheme previously detailed. The chain run 200,000 times after a burn-in phase of 100,000 iterations. Simulations are recorded every ten iterations.

Figure 1 displays prior and posterior distributions of periodicity and amplitude of GDP cycles in both US and EMU. It can be seen that the periodicity posterior mode is about 7 years for US and about 9 years for EMU. For both US and the EMU, the posterior mode is a bit larger than that of the prior. Maybe this reflects the relatively long expansion of the 1990's. There seems to be more precision around the periodicity of cycles in the US than in the EMU, but this is mainly due to the difference between priors: using the EU prior for US data yields analogous dispersion. More precision is obtained for the cycle amplitude parameter. With a mode at 0.8 and a comparable dispersion, the posterior distributions of US and EMU cycle amplitude seem much alike. This suggests that a similar number of quarters is necessary to both economies for absorbing a demand shock.

Figure 2 shows the prior and posterior distributions of the correlation coefficient $\rho_{c\pi}$, of the inverse signal to noise ratio V_c/V_p and of the output mean growth rate μ_p . For US, the inverse signal to ratio reported is that obtained in the second sample-period, i.e. $\alpha_c V_c / (\alpha_p V_p)$. The posterior distribution of $\rho_{c\pi}$ is highly concentrated around a mode of about 0.15 for US and about 0.25 for EMU. For both US and EMU, the variance of the

shocks in the gap is roughly one-third the variance of the long-term shocks, and this is obtained with a good accuracy: shocks on potential output are thus dominating these two GDP series. Hence the pile-up problem mentioned in introduction does not occur. Figure 2 also shows the distribution of the output mean growth rate: the posterior modes imply an average annual growth rate of about 3.0% for US and 2.4% for EMU.

Figure 3 displays the prior and posterior densities of the scale parameters α 's. The trend and cycle innovations variances seem both reduced by a factor of about 0.2, letting the signal to noise ratio almost constant. The moderation of the inflation shocks variance is less pronounced. The posterior precision around the scale parameters confirms the relevance of the variance break specification for US GDP and inflation.

Figure 4 presents the prior and posterior distributions of the impact multipliers β and λ . The posterior distributions put small weight in the neighbourhood of zero so the data seems to validate the prior restriction on positive responses. More constancy can be seen in the posterior response of inflation to shocks on the gap than to shocks on output growth. Since gap and output growth mainly differ in the periodicity of their movements, it may be the noise amplification that the differencing operation implies that has obscured the empirical link with inflation. This result suggests that Phillips curve regressions can be sensitive to the dynamic properties of the proxy used for describing detrended output.

Figure 5 shows the dynamic response of inflation to a shock on the gap with associated 90% highest posterior density region (HPD; see Box and Tiao, 1973, Chap. 2.8). This impulse response function is obtained as the posterior mean of the polynomial $\{\beta + \lambda(1 - L) + \kappa_\pi \phi_c(L)\} / \{\phi_c(L) \phi_\pi(L)\}$. The total inflationary effect of a shock on the gap is then $\{\beta + \kappa_\pi \phi_c(1)\} / \{\phi_c(1) \phi_\pi(1)\}$; its posterior distribution is displayed in Figure 6, together with the distribution implied by the parameters priors. It can be seen that, besides positiveness, we have not much constrained the shape of the response. The dynamics of the propagation of a shock on the gap is similar in US and in EMU: after one year the inflationary effects are not significantly different from zero at the 10% level. Besides the contemporaneous reaction related to the correlation parameter, the largest impact on inflation of a shock on the gap occurs with a three-quarter delay. The total response of inflation is slightly stronger in the euro area than in the US: according to posterior modes, a 1 percentage-point rise of the gap in one quarter yields an inflation pressure of about 0.45 points for EMU against 0.38 for US.

Figure 7 shows the posterior mean of the gap all along the sample together with the maximum likelihood (ML) estimates. The sequence of turning points suggests a delay in the fluctuations of the EMU economy with respect to the US. Figure 7 also displays the 90% HPD interval and the 90% ML confidence bands computed with the Ansley and Kohn (1986) procedure that accounts for parameter uncertainty. The effect of the US variance break is quite evident: the confidence interval shrinks by one-half in the second period. For EMU and for US in the last period, ML and Bayesian analysis yield a similar accuracy, with a slight advantage to the Bayesian approach.

6. Conclusion

In this paper we have developed a Bayesian analysis of Kuttner's bivariate Phillips-curve model for estimating output gaps. This methodology enables potential output and NAIRU estimates to incorporate prior information. Given the amount of macroeconomic

knowledge and business cycle studies available, we believe that this Bayesian framework presents some appeals for practitioners. We have made it available into Program GAP, version 4.0, that can be downloaded at www.jrc.cec.eu.int/uasa/prj-gap.asp, following the link Further Information.

The results show that the data agree with Phillips-curve theory. For both EMU and US, we could observe a sharp response of inflation to the gap and a more volatile reaction to output growth. We also obtained response functions of EMU and US inflation to shocks to the gap with a similar dynamic. EMU inflation seems however slightly more reactive to shocks on the gap. The posterior distribution of the cycle periodicity never appear very precise, a point that would deserve a focused investigation. Given our priors, the gap posterior mean was found in close agreement with the ML estimates. Yet the gain in accuracy achieved with the Bayesian approach was noticeable, showing that prior information can help in characterizing unobserved patterns when evidence in the data is only moderate.

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Figure 1: Densities of cycle periodicity and amplitude

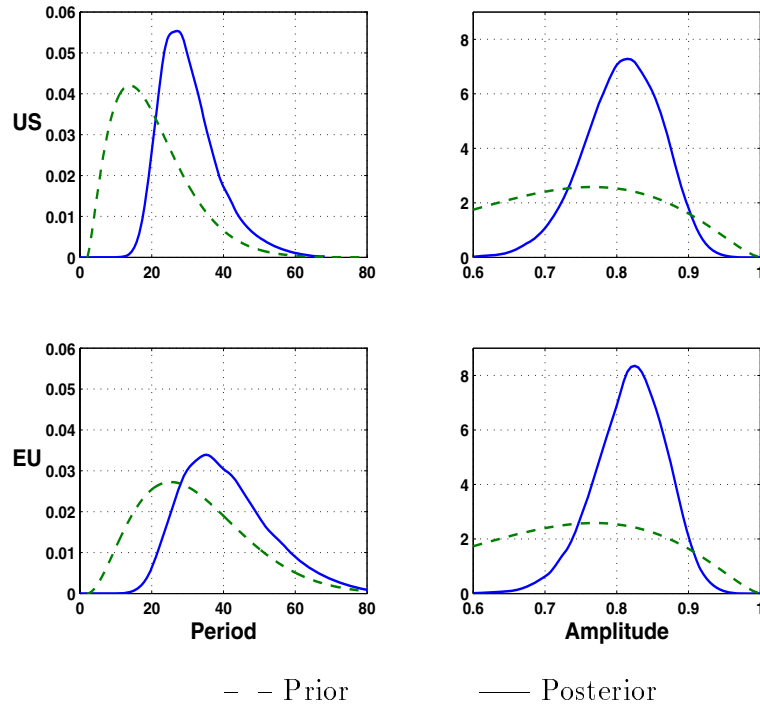


Figure 2: Densities of correlation between innovations in gap and in inflation, of inverse signal to noise ratio and of output mean growth rate

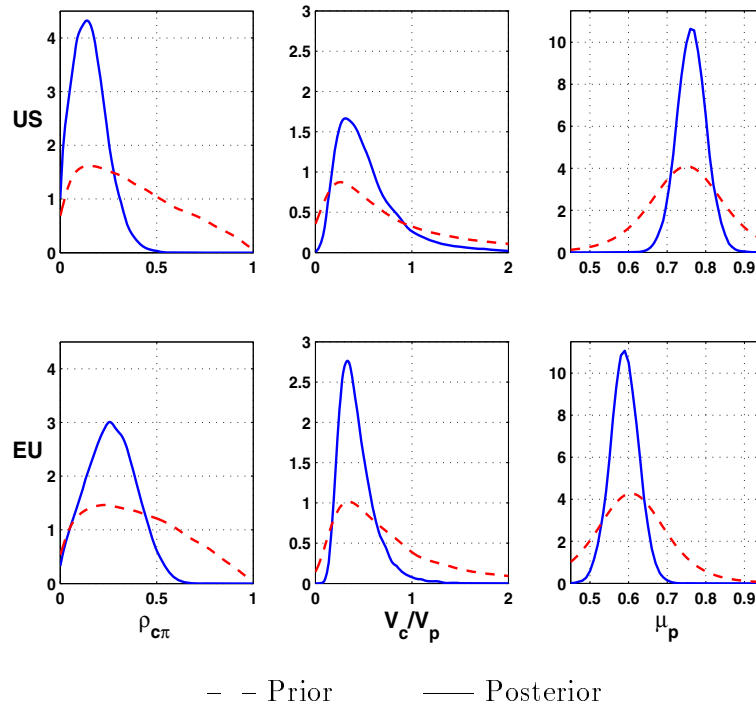


Figure 3: Densities of scale factors

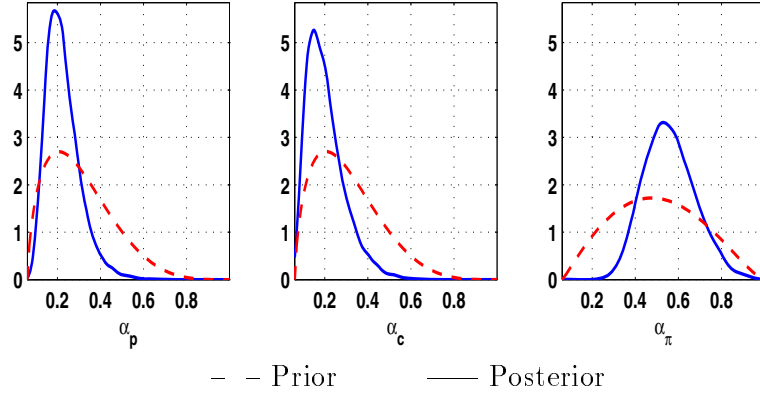


Figure 4: Densities of impact multipliers

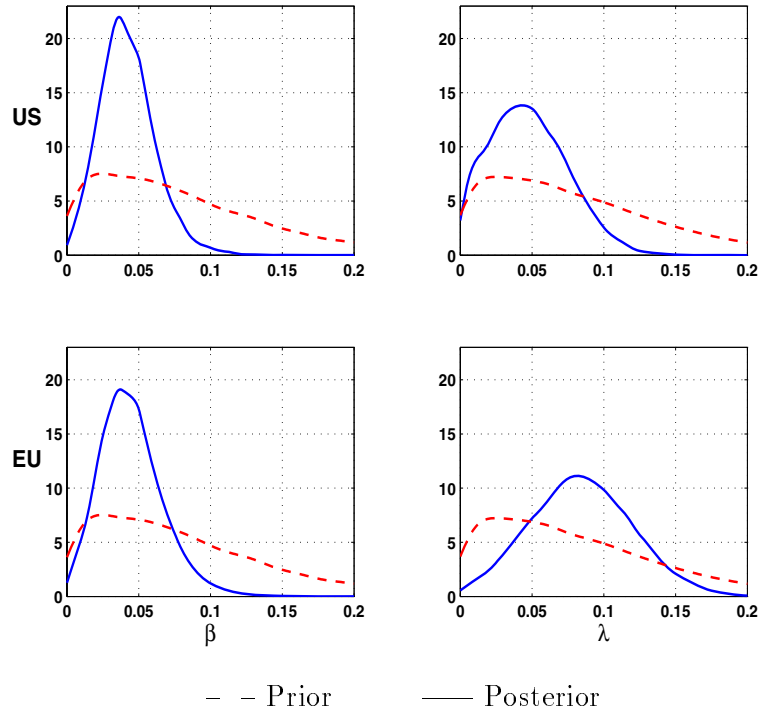


Figure 5: Average response of change in inflation to a shock on output gap with 90% HPD region

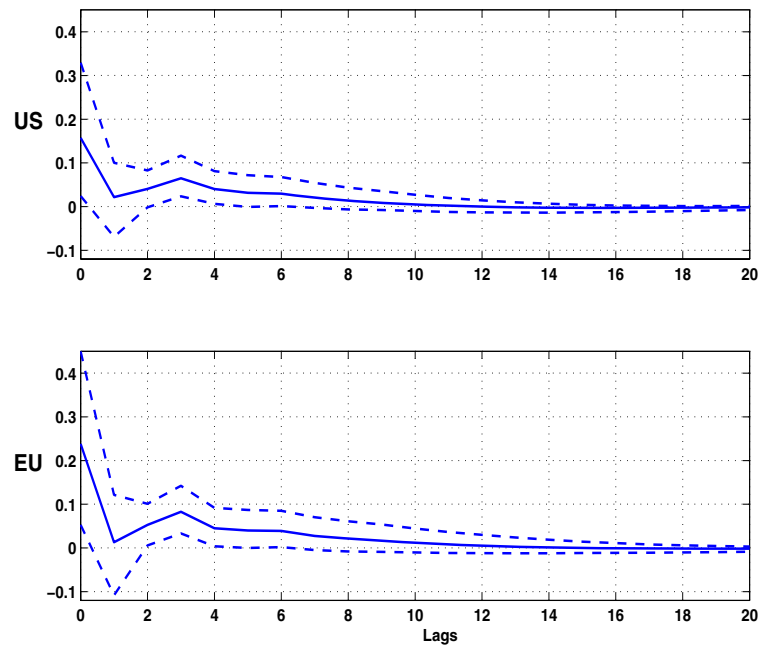


Figure 6: Posterior density of total response of change in inflation to a shock on output gap

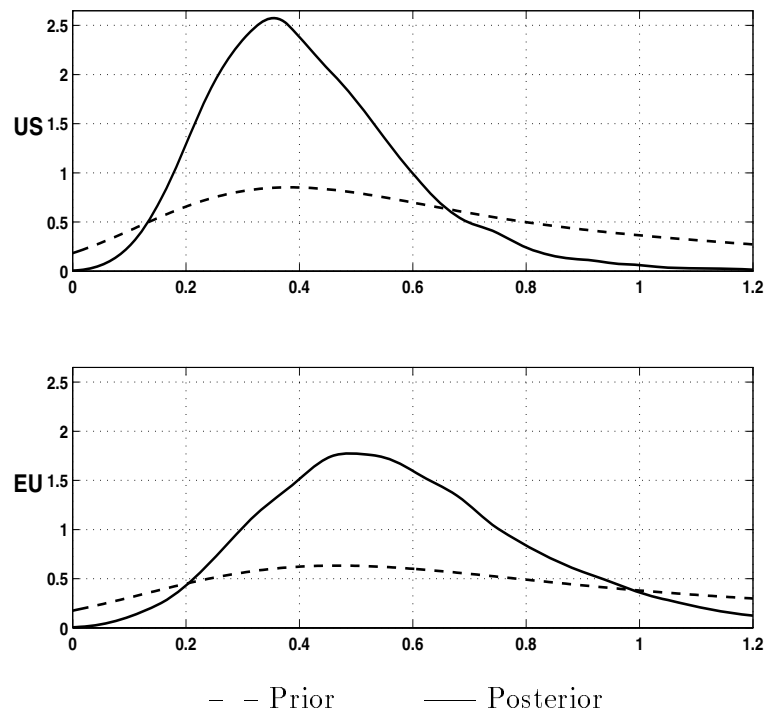


Figure 7: Gap posterior mean (—) with 90% HPD region
ML estimate (- -) with 90% confidence bands

